

## Magnetic Structure of the Low Temperature Antiferromagnetic Phase of $V_2O_3$ \*

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Using a magnetic space group analysis,  $V_2O_3$  has been shown to have the  $(A_B)_a-G_b-(A_B)_c$ -type spin structure, which must be explained by both competitive metal-metal and metal-oxygen interactions. The spin orientation is analyzed in terms of the contribution provided by the magnetic anisotropy energy in  $V_2O_3$ .

### Introduction

Numerous investigations (1) have shown that a sharp semiconductor-metal transition occurs in  $V_2O_3$  in the temperature region 150–170°K and that an anomalous change in physical properties of  $V_2O_3$  is encountered over the broad temperature range of 350–600°K. The low temperature transition is accompanied by a crystallographic change from monoclinic to rhombohedral symmetry (2). By contrast, the high temperature transition proceeds without change of the rhombohedral lattice symmetry (3). Relatively little attention has been devoted to the low temperature transition and to lattice properties of the monoclinic  $V_2O_3$  phase. Earlier investigations involving neutron diffraction (4), magnetic susceptibility studies (5), Mössbauer measurements (6), and nuclear magnetic resonance (7), yielded conflicting evidence on whether monoclinic  $V_2O_3$  exhibits magnetic spin order. However, the work by Moon (8) and Heidemann (9), using spin-flip polarized neutron scattering techniques, showed conclusively that all V atoms in planes perpen-

dicular to the  $[010]_m$  direction of the monoclinic phase are ferromagnetically aligned, while adjacent planes are coupled antiferromagnetically. No attempts appear to have been made to interpret the spin structure of the low temperature phase on the basis of phenomenological considerations; we have therefore undertaken this task.

### Spin Configuration as Interpreted by Magnetic Space Groups

The following analysis is based on the procedure by Bertaut (10) for the magnetic space group appropriate to monoclinic  $V_2O_3$ . Dernier and Marezio (11) have reported the crystal structure and atomic positions of the low temperature, monoclinic, antiferromagnetic phase (Fig. 2) using the space group  $I2/a$  suggested by McWhan and Remeika (12). Aside from the identity, the following independent symmetry operations of  $I2/a$  superpose the crystal on itself: (i) rotations about the twofold axis  $2b$  ( $a = \frac{1}{2}, c = \frac{1}{2}$ ); (ii) inversion about the origin  $i$  ( $a = 0, b = 0, c = 0$ ); and (iii) translation-rotation about the twofold screw axis  $\tilde{2}b$  ( $a = \frac{1}{2}, c = \frac{1}{2}$ ). Other, equivalent symmetry operations could have

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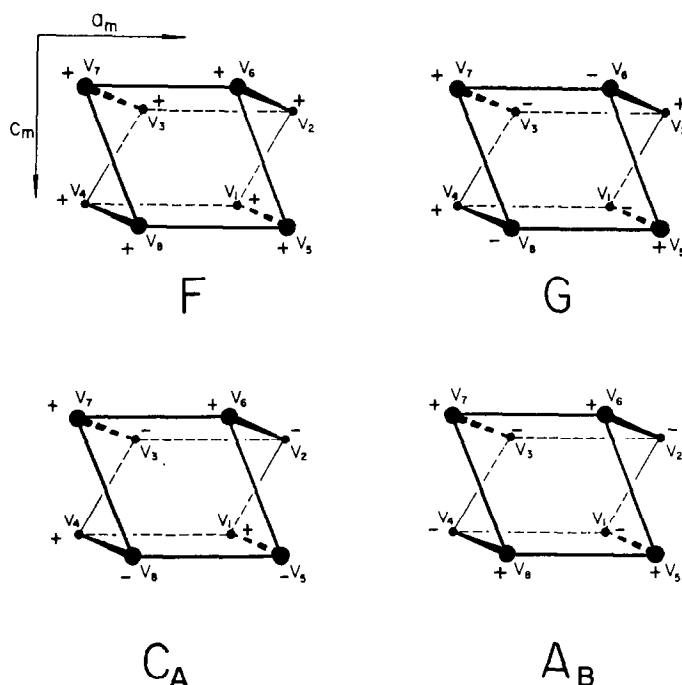


FIG. 1. Conventional basis configurations for spin alignments in monoclinic  $V_2O_3$ . The geometric figures indicate relative  $V_i$  positions. The + and - signs are to be interpreted in terms of components, as explained in the Text. The figures shown here represent one of several sets for different spin-up (+) and spin-down (-) assignments at the indicated vertices. For  $C_B$ ,  $C_C$  configurations the linear chains run along the  $b_m$  and  $c_m$  unit cell axes, respectively, where  $b_m$  is perpendicular to both  $a_m$  and  $c_m$ . For  $A_A$ ,  $A_C$  configurations the ferromagnetic planes are bounded by the  $(b_m c_m)$  and  $(a_m b_m)$  axes, respectively. Front face,  $(020)_m$  plane, contains atoms  $V_5$ - $V_8$ ; rear face,  $(010)_m$  plane, contains atoms  $V_1$ - $V_4$ .

been used, but those introduced above are convenient.

The "basis functions" on which the above operations are to be performed are the conventional configurations  $F$ ,  $G$ ,  $C$ , and  $A$  (10), one set of which is shown in Fig. 1. These four groupings refer, respectively, to the ferromagnetic, nearest neighbor antiferromagnetic, linear ferromagnetic, and layer-type antiferromagnetic-type spin alignments on the lattice (in this case, vanadium) sites. The vertices in the front face of Fig. 1 represent vanadium sites (not vertices of the unit cell) in the  $(020)_m$  plane of  $V_2O_3$ ; those on the back face are V sites in the  $(010)_m$  plane of  $V_2O_3$ . Attention is directed to the numbering of the V units which is indicated in Fig. 2 and explained in the caption of Fig. 1.

Eight distinct configurations of spin arrangements must be considered: those shown in Fig. 1 and additional sets ( $C_B$ ,  $C_C$ ,  $A_A$ ,  $A_C$ ) which correspond to different dispositions of "up" and "down" spins on the vertices of the  $C$  and  $A$  configurations as explained in the figure caption. All of these are to be understood as applying to the spin components. For example, the representation for site  $V_2$  in Fig. 1 indicates that for the  $F$  and  $G$  configurations the  $V_2$  atom on that site has positive spin components directed along the  $\hat{a}_m$ ,  $\hat{b}_m$ ,  $\hat{c}_m$  directions while for the configurations  $C_A$  and  $A_B$ ,  $V_2$  has negative spin components along these axes.

One now operates on each of the magnetic symmetry structural units (basis functions) with each of the symmetry operations men-

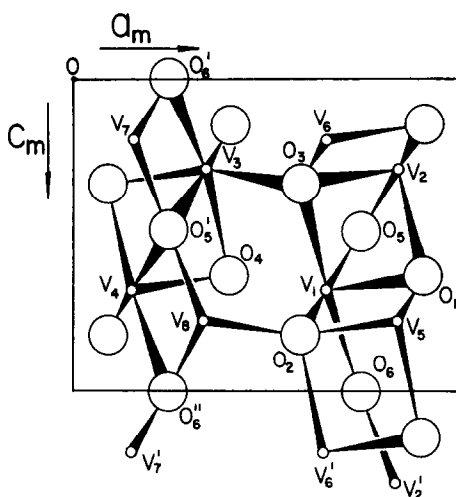


FIG. 2. A portion of the unit cell of monoclinic  $V_2O_3$ .  $V_1$ - $V_4$  are in the  $(010)_m$  plane,  $V_5$ - $V_8$  in the  $(020)_m$  plane. Note the numbering scheme.  $O$  designates the origin relative to which the symmetry operations are carried out. Small circles represent V atoms, large circles, O atoms.

tioned earlier, thereby interchanging spins on various equivalent sites. As expected, these interchange operations either leave a given spin component on equivalent sites unaltered, or else a sign change occurs. The results of these operations are displayed in Table I: here the entries + or - show that under the operation and for the component indicated in the heading, the spins remain unaltered or reverse sign. For example, when the operation

TABLE I

RESULTS OF SYMMETRY OPERATIONS ON THE BASIS FUNCTION COMPONENTS

Vector	Operation $2b$			Operation $i$			Operation $\bar{2}b$		
	$a$	$b$	$c$	$a$	$b$	$c$	$a$	$b$	$c$
$F$	-	+	-	+	+	+	-	+	-
$G$	+	-	+	+	+	+	-	+	-
$C_A$	+	-	+	+	-	-	-	+	+
$C_B$	+	-	+	+	+	+	+	-	+
$C_C$	-	+	-	-	-	-	-	+	-
$A_A$	-	+	-	-	-	-	+	-	+
$A_B$	-	+	-	+	+	+	+	-	+
$A_C$	+	-	+	-	-	-	-	+	-

$2b$  is performed on configuration  $F$ , spin components along  $a$  and  $c$  change in sign, but those along  $b$  do not; hence, the - + - sequence on the top left of the Table. In arriving at the results of Table I, it should be noted that spin angular momentum is represented as a pseudovector  $\mathbf{r} \times \mathbf{p}$ ; hence, the inversion process does not change spin direction, whereas rotation about an axis perpendicular to the spin alignment does.

In Table II are summarized the eight sets of representations  $\Gamma_i$  appropriate to the transformation of the basis functions, under the symmetry operations described earlier. This is equivalent to the problem of distributing + and - signs among the operations  $2b$ ,  $i$ ,  $\bar{2}b$ . For example, for  $\Gamma_6$ , the (- + -) designations refer in that order to the overall results of the  $2b$ ,  $i$ , and  $\bar{2}b$  operations. This sequence is matched by the spin components along  $a$  of the basis function  $F$ ; i.e.  $F_a$ . As seen from Table I, when the  $2b$ ,  $i$ ,  $\bar{2}b$  operations are carried out on the  $a$  component of the  $F$  magnetic symmetry structure, one recovers the values  $-F_a$ ,  $+F_a$ ,  $-F_a$ , respectively. No other basis function with spin components along  $a$  transforms in this particular manner, although the  $C_B$  basis with spin components along  $b$ ,  $(C_B)_b$ , and the  $F$  basis with components along  $c$ ,  $F_c$ , do. Other entries in Table II are constructed similarly; the corresponding magnetic ordering may (but need not necessarily) occur. Table II summarizes all possible spin orderings that could be encountered in any monoclinic lattice of

TABLE II

REPRESENTATIONS OF BASIS FUNCTION COMPONENTS

	$a$	$b$	$c$
$\Gamma_1(+++)$	$(C_B)_a$	$F_b$	$(C_B)_c$
$\Gamma_2(++-)$	$G_a$	$(A_B)_b$	$G_c$
$\Gamma_3(+++)$	$(C_A)_a$	$(C_C)_b$	$(C_A)_c$
$\Gamma_4(-++)$	$(A_B)_a$	$G_b$	$(A_B)_c$
$\Gamma_5(+--+)$	$(A_C)_a$	$(A_A)_b$	$(A_C)_c$
$\Gamma_6(-+-)$	$F_a$	$(C_B)_b$	$F_c$
$\Gamma_7(-+-)$	$(A_A)_a$	$(A_C)_b$	$(A_A)_c$
$\Gamma_8(---)$	$(C_C)_a$	$(C_A)_b$	$(C_C)_c$

$I2/a$  symmetry. As far as is known to the writers such a tabulation has not been constructed before.

Neutron diffraction studies (8, 9) have established that  $V_2O_3$  is a layer-type antiferromagnet whose spins are ferromagnetically aligned in the  $(020)_m$  planes, which themselves are antiferromagnetically coupled. According to Table II and Fig. 1 the appropriate representations for such a configuration can only be  $\Gamma_4(- + +)$ , which involves the  $(A_B)_a - G_b - (A_B)_c$ -type spin alignment. Three-dimensional ferromagnetic ordering is precluded in  $V_2O_3$  because the  $F_i$  components in Table II occur only in conjunction with basis functions other than the  $A$  type. The  $G_b$ -type ordering which is permitted by the  $\Gamma_4(- + +)$  representation is, in fact, consistent with Moon's data (8), though he obtained as good a fit to his data without requiring the V spins to have any components along the  $b$  axis. Thus,  $G$ -type spin ordering along the  $b$  direction, if present, is weak.

### Indirect Exchange Interaction

The mechanism of the superexchange interaction was first proposed by Kramers (13); Anderson (14) investigated the details of this process. Semiempirical rules for the superexchange interaction effects have been discussed by Goodenough (15) and enunciated by Kanamori (16). An attempt was made to apply these concepts to the case of monoclinic  $V_2O_3$ .

The spin Hamiltonian for  $V_2O_3$  may be written (17) as follows:

$$H = \sum_{j>k} \{J_{jk} S_j \cdot S_k + S_j \cdot K_{jk} \cdot S_k\} + \sum_j DS_{jz}^2 \quad (1)$$

The first term represents the superexchange interaction, the second term accounts for the dipolar effects, and the third term represents the single-ion anisotropy due to spin-orbit interactions.

We first attempted to rationalize the observed spin ordering in monoclinic  $V_2O_3$  on the

basis of the first term, with  $J_{jk} = -b^2 J(p, d)/\Delta E^2$ , where  $b$  is the transfer integral connecting a vanadium atom to the nearest neighbor oxygen,  $\Delta E$  is the energy difference involved in the electron transfer, and  $J(p, d)$  is the appropriate exchange integral. We found it impossible to force the observed antiferromagnetic coupling of ferromagnetic layers in  $V_2O_3$  by this mechanism. This most probably indicates that overlap between vanadium  $e_x$  and  $a_1$  orbitals with oxygen  $p$  orbitals is too large in  $V_2O_3$  for second-order perturbation theory to be applicable, and further, that these contributions are in competition with the direct metal-metal interactions discussed by Goodenough (18) and reviewed elsewhere (19). Thus, we adopt Goodenough's explanation of antiferromagnetic ordering (18), with the modification that the energy levels, disposition of bands, and their widths are strongly affected by metal-oxygen interactions which occur in parallel with the direct metal-metal bonding. A quantitative treatment of this type for monoclinic  $V_2O_3$  has recently been supplied by Ashkenazi and Weger (20) and by Castellani *et al.* (21), on the basis of detailed band structure calculations, in which electron correlations and polarizations are taken into account. These analyses show that in the vicinity of the Fermi level the valence bands are of the order of 0.8 eV in width, separated by a gap of approximately 0.2 eV from higher lying conduction bands. In these circumstances the simple superexchange formalism cannot be expected to apply. These findings support earlier speculations (22), based on thermoelectric power measurements, that the monoclinic phase is an insulator characterized by an itinerant rather than a localized charge carrier regime.

### Magnetic Interactions

Both the experimental work and the theoretical analysis shows that the spin moments of vanadium are lying within  $a_m - c_m$  planes of the monoclinic phase of  $V_2O_3$ . We

seek to interpret the orientation of the spins in these planes relative to the short V-V distance in the (010)<sub>m</sub> planes [for example, V<sub>1</sub>-V<sub>2</sub> in Fig. 2] which defines the hexagonal c<sub>H</sub> axis direction in rhombohedral V<sub>2</sub>O<sub>3</sub>. In conformity with general practice we utilize the last two terms in Eq. (1) to determine the spin orientations in an itinerant antiferromagnet.

We turn first to the central dipolar interaction term which may be rewritten as

$$E_d = - \sum_j \frac{1}{r_{ij}^3} \left\{ m_i \cdot m_j - 3 \frac{(m_i \cdot r_{ij})(m_j \cdot r_{ij})}{r_{ij}^2} \right\}, \quad (2)$$

where  $r_{ij}$  is the separation between V atoms located on sites  $i$  and  $j$  and  $m_i$  is the dipole moment associated with the spin on site  $i$ .

In what follows we wish merely to demonstrate that this contribution is small compared to the anisotropy energy associated with the last term in Eq. (1). For this purpose it suffices to restrict the summation in Eq. (2) to nearest neighbors of atom V<sub>*i*</sub>. This yields the approximate value

$$E_d \approx -0.048 \cos 2\theta - 0.039 \sin 2\theta \text{ (cm}^{-1}\text{)} \quad (3)$$

where  $\theta$  is the canting angle of the spin in the  $a_m$ - $c_m$  plane relative to the former hexagonal  $c$  axis. Extension of the summation to second, third, and higher order neighbor shells is necessary to obtain a more exact expression, but it is very unlikely that the results so obtained would alter the qualitative conclusions reached below.

We next turn to the third term in Eq. (1). When the anisotropic energy term is reexpressed in terms of  $\theta$ , one obtains

$$E_a = DS(S - \frac{1}{2}) \cos^2 \theta \approx 4.17 \cos^2 \theta \text{ (cm}^{-1}\text{)} \quad (4)$$

in which the value  $D = 8.34 \text{ cm}^{-1}$ , obtained from paramagnetic resonance measurements (23) on V-doped Al<sub>2</sub>O<sub>3</sub>, was introduced. The above can serve only as an order-of-magnitude estimate for  $E_a$ , but is probably correct to the extent of showing that  $E_a$  significantly exceeds  $E_d$ . As a first approximation, one may thus

assume that the spin direction in the  $a_m$ - $c_m$  plane is determined by the dependence of  $E_a$  on  $\theta$ , which exhibits a minimum in  $E_a$  for  $\theta$  at right angles to the  $c_H$  axis. Experimentally, a canting angle of 71° has been reported (8). Thus, the basic features of the experimental investigations on spin alignments in V<sub>2</sub>O<sub>3</sub> have been reasonably well accounted for, but a more sophisticated analysis would be required for a refined interpretation.

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